Original scientific paper *UDC 793.3.071.2:159.923.2.072.59*

Differences in the level of musical abilities of male and female dancers

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1. Introduction

Students of sport and physical education need to be introduced to the basic elements of music because they have both educational and incentive role. The culture of movement accompanied by music has a beneficial effect on the body, thoughts and spirit. The connection between the auditory and the visual in the form of movement is necessary for teaching dance, rhythmic gymnastics, sports gymnastics, synchronized swimming and so on.

Music is the art which uses tones as means of its expression. It extends to our conscience, with the help of sense of hearing. Therefore, it may be concluded that music is apprehended by listening. In music, enjoying property is estimated according to the nature of the feeling it produces. To feel and to understand are not the synonyms. A person should feel in order to understand.

Above all, each individual has to feel his or her body in order to perform a beautiful movment, and each movement is beautiful if it is musically performed.

The human body should be conceived as the most perfect musical instrument. Music is the art of permanent motion, since it is composed of tones which are themselves undulation, motion. For this reason it is capable of making us move. Music drives us to the dance. We can, for instance, march to the sounds of music cheerfully, vigorously. Besides its force to make us dance or march, music has the greatest ability of all arts to evoke various feelings and moods. While listening to music, we are capable of sensing grief and pain, sadness or happiness, we cry or laugh, being full of sorrow or vigor.

Music is capable of imitating the movements in nature by its artistic means. A composer may, with the help of musical tones, present water gurgling, storm, wind, drizzling, forest murmur, buzzing of bees, ringing of bells etc. This is not music, but if it is expressed in musical tones, it gets the full artistic value.

In order to mark as accurately as possible a special character of some composition as a whole or its part, many diverse expressions are applied. Those expressions could be utilized either independently or combined with a tempo mark, and are indicated at the beginning of a composition or even in its further course (excitedly, restlessly, gracefully, tenderly, enthusiastically, passionately, brightly, facetiously, quietly, melodiously, affectionately, warmly, firmly, distinctly, painfully, expressively, wildly, solemnly, proudly, sadly, relaxed, grandly, nobly, heavily, willingly, freely, rapidly, simply, emotionally, cheerfully, in a flying manner, noisily, gently, extinguished, awakened, tempestuously, piously etc.

2. The methods of research

2. 1. The sample of examinees

The sample of examinees is conditioned by the financial capabilities which are necessary for conducting the research procedure. Nevertheless, the sample depends on the number of qualified and trained measurers, on the instruments and standardized conditions under which the planned research should be realized.

In order to conduct the research correctly and to provide sufficiently stable results, in the terms of the sampling error, it is required to hire a satisfactory number of examinees for the sample. Most of the samples for this type of research should be conditioned by the aims and tasks of the research, as well by the size of population and the degree of variability of the applied system of parameters.[7]

According to the selected statistical-mathematical model and the aim of research, the sample of examinees includes 131 female dancers and 136 male dancers, aged from 11 to 13, that are actively involved in standard and Latin American dances in the Serbian dancing clubs.

The size of the so determined sample should satisfy the following criteria:

- The effectives of the sample should be planned so that it allows as many degrees of freedom as necessary for any coefficient in the pattern or correlation matrix, which is equal to or bigger than 0.22, to be considered as different from zero with an inference error less than 0.01.
- In order to successfully apply the adequate statistical methods based on the most recent convictions, the number of subjects in the sample must be five times larger than the number of the applied variables.

During all the factor procedures, it should always be kept in mind that the results of the analysis depend on three major systems that determine the selection and transformation of information: the sample of variables, the sample of examinees and the selected extraction, or rotation, method.[8]

2. 2. The sample of variables

The evaluation of musical abilities has been accomplished on the basis of the well-known Seashore test battery that estimates musicality. This test lasts for 30 minutes and it consists of 6 groups of tasks that are listened to from an audiotape, and the answers are noted on the prepared answer sheets for that purpose. Auditory is provided by the regular schedule of the sound system and the volume so that all the examinees could be put under the same experimental conditions.

This test estimates the following dimensions:

- Pitch discrimination test: it consists of five columns, and each column contains ten tasks. For each task two tones are played. The examinee is

7 Popović, D.: Determining the structure of psychosomatic dimensions in fights and developing the procedures for their evaluation and monitoring - The Monograph, the Faculty of Physical Education, University of Priština, Priština, 1993.

8 Popović, D.: Determining the structure of psychosomatic dimensions in fights and developing the procedures for their evaluation and monitoring - The Monograph, the Faculty of Physical Education, University of Priština, Priština, 1993.

to determine whether the second tone was higher or lower than the first one.

- Tone intensity discrimination: it consists of five columns. Each column contains ten tasks. For each task two tones are played. The examinee is to determine whether the second tone was louder or quieter than the first one.
- Rhythm recognition test: it consists of three columns. Each column contains ten tasks. For each task two rhythmical structures are played. The examinee is to determine whether the second rhythmical structure was the same or different from the first one.
- Tone duration discrimination test: it consists of five columns. Each column contains ten tasks. For each task two tones with different duration are played. The examinee is to determine whether the second tone was longer or shorter than the first one.
- Timbre discrimination test: it consists of five columns, and each column contains ten tasks. For each task two tones are played. The examinee is to determine whether the second tone was the same or different from the first one.
- Tonal memory test: it consists of three columns. Each column contains ten tasks. On the column A for each task two melodies are played three times. On the column B two melodies of four tones are played, and on the column C two melodies of five tones are played. The examinee is to determine for each task in which tone the second played melody differs from the first one. For the column A: the first, second or third tone, for the column B: the first, second, third or fourth tone and for the column C: the first, second, third, fourth or fifth tone.

The evaluation is carried out so that each correct answer for each task in all the tests is worth one point. The total sum of points scored in particular tasks of each test separately, constitutes the result. The result expressed in points should be converted to percentages. The female examinees, according to the number of points obtained on particular tests, depending on their age, are classified in certain classes from "A" to "E".

2. 3. The methods for data processing

In the discussion on the results of an empirical research (Vučić, Vukmirović, Vukmirović i Radojičić, 1997), presented at the tenth meeting of the Section for classifications of the Union of Statistical society of Yugoslavia, PhD Boris Wolf warned that the results obtained by the canonical discriminant analysis were absurd, because despite the canonical correlations of 0.99 and 0.85 the structural vectors of both discriminant functions were practically the null vectors. It took only a few minutes to determine that it was due to the fact that the analysis was performed by the program Discriminant... from the program package SPSS which explicitly defines the structure of discriminant factors as a matrix of cross-correlations between the variables, from which the effects of belonging to a group, that were the subject of the analysis and discriminant functions defined according to the standard formulation of canonical discriminant problem, were partialized. Since such an implementation of canonical discriminant analysis is typical of many, but, fortunately, not all the statistical program packages or systems, in this study, after the reformulation of canonical discriminant analysis, derived under the model of canonical correlation analysis, it was demonstrated that the implementation of the standard definition of a discriminant model had caused, in some marginal cases, insuperable numerical difficulties, and that the definition of the structure of canonical factors, which followed from the standard discriminant model, was completely absurd, because the components of the variables, according to which the discriminant functions were formed, had no influence on the structure of the so determined discriminant factors. This is the reason for applying the algorithm and program for canonical discriminant analysis from the program system SAS.

DEFINITIONS

Let

$$E = \{e_i; i = 1,..., n\} \subseteq P = U_p^g P_p \mid P_p \cap P_q = O, p \neq q$$

be a random sample from some heterogeneous population of objects that consists of g subpopulations P_p and let

$$W = \{w_p; p = 1,..., g\}$$

be a nominal variable whose categories w_p define the obligatory and unique properties of the objects from the subpopulations P_p.

Let

$$V = \{v_j; j = 1,..., m\} \subseteq U$$

be a set of qualitative and quantitative variables that are multivariately normally distributed in each subpopulation P_p from P selected so that they represent a universe of variables U defined by some consistent and operationalizable theory about the behaviour of the objects from P.

Let $e = (e_i)$, $i = 1,...,n$: $e_i = 1 \ \forall \ e_i$ be a summing vector of rank n. Let

$$Z = E \otimes V \mid Z^t e = 0, \ \mathrm{diag}\ (Z^t Z) = I$$

be a data matrix in the standard normal metrics obtained by the description of the set E on the set V, and let

$$S = (s_{ip}) = E \otimes W$$

be the indicatory matrix whose elements s_{ip}, $i = 1,...,n$; $p = 1,..., g$ are defined by the function

$$\{s_{ip} = 1 \mid e_i \in w_p,\ s_{ip} = 0 \mid e_i \notin w_p\}.$$

Let

$$R = Z^t Z$$

be the matrix by which, based on the maximum likelihood criterion, the intercorrelations of variables from V are evaluated; assume that matrix is nonsingular and mark the regular inverse of that the matrix with R^{-1}.

Let

$$P = S(S^t S)^{-1} S^t$$

be a projector into the hypercube defined by the vectors s_p from S, and let

$$Q = I - P$$

be a projector into the hypercube that is orthogonal to the hypercube defined by the vectors s_p from S because, certainly, $PQ = 0$.

Let

$$G = PZ$$

be a matrix obtained by the projection of the vectors z_j from Z into the hypercube defined by the vectors s_p from S, and let

$$H = QZ = Z - PZ$$

be a matrix obtained by the projection of the vectors z_j from Z into the hypercube that is orthogonal to the hypercube defined by the vectors s_p from S.

The covariance matrix of the variables from G will be

$$A = G^t G = Z^t P Z;$$

we find that the matrix A is, concurrently, the cross co-variance matrix of the variables from Z and G.

The covariance matrix of the variables from H will be

$$W = H^tH = Z^tQZ = R - A;$$

It's evident that the matrix W, is concurrently, the cross-covariance matrix of the variables from Z and H, and the intercorrelation matrix of the variables from Z may be decomposed so that

$$R = A + W.$$

Let

$$\Lambda = (\lambda_j) = \text{diag } W$$

and let

$$H^2 = (\eta_j^2) = \text{diag } A = I - \Lambda.$$

It could be easily shown (Guttman, 1988; Momirović, 1989; Momirović and Zorić, 1996) that, in this metrics, the elements λ_j of the matrix Λ are actually Wilks' measures of the relative intragroup dispersion and therefore the elements η_j^2 of the matrix H^2 are the squares of Fisher's intergroup correlation coefficients, so it is possible to reformulate Rao's (Rao, 1948; 1975) method of canonical discriminant analysis in the manner that makes the sense of the discriminant factors and structural matrices of the discriminant factors usually applied to indentify the content of those functions much clearer.[9]

RAO'S METHOD OF CANONICAL DISCRIMINANT ANALYSIS

The method known as canonical discriminant analysis (Rao, 1948; 1952; 1968; 1973; Rao and Slater, 1949) may be defined in a many different, but basically equivalent ways (Anderson, 1966; Anderson, 1984; Bryan, 1951; 1975; Cooley and Lohnes, 1971; Glahn, 1968; Hadžigalić, 1984; Hadžigalić, Bogdanović, Tenjović and Wolf, 1994; Ivanović, 1963; 1977; Kendall and Stuart, 1976; Kovačić, 1994; Momirović, Gredelj and Szirovicza, 1977; Momirović and Dobrić, 1984; Momirović, Knežević, Kuzeljević and Radović, 1994; Momirović and Zorić, 1996; Mulaik, 1972; Romeder, 1973). Although, it is most frequently deduced as a generalisation of multivariate analysis of variance (Rao, 1948; 1952; 1968; 1973; Rao and Slater, 1949; Anderson, 1966; Anderson, 1984; Bryan, 1951; 1975; Cooley and Lohnes, 1971; Kendall and Stuart, 1976; Kovačić, 1994; Momirović, Gredelj and Szirovicza, 1977; Romeder, 1973), yet it is treated or di-

9 This will be performed in the manner similar but not identical to the one proposed in the works of Hadzigalić (1984), Momirović and V. Dobrić (1984) and Momirović and Zorić (1996). The modification of their deductions was made so that the level of absurdity of some implementations of canonical discriminant analysis in commercial statistical program products be clearer, as well as the dangers to which those who blindly apply the program products are exposed.

rectly performed as a special case of canonical correlation analysis (Glahn, 1968; Anderson, 1984; Hadžigalić, Bogdanović, Tenjović and Wolf, 1994; Momirović, Knežević, Kuzeljević and Radović, 1994; Momirović and Zorić, 1996) or as a special case of a component model of factor analysis (Mulaik, 1972; Hadžigalić, 1984; Momirović and Dobrić, 1984); in a special case when g = 2, famous as Fisher's case, it may be derived as a special case of regression analysis.

Although, under some conditions, all those manners are equivalent for the evaluation of canonical correlation coefficients, this is not so for the definition of discriminant functions and identification structures associated with those functions; and since the deduction which is based on the generalization of the variance analysis assumes that the condition the covariance matrices of variables in subpopulations, which should be discriminated, are identical, is also fulfilled, what is rather an exception than the rule, here will be proposed a reformulation of canonical discriminant analysis that treats the method as a special case of Hotelling's model of biorthogonal canonical correlation analysis (Hotelling, 1936) which follows the main lines of the reformulation of the method suggested by Momirović and Zorić (1996).[10]

Let B be an unknown matrix of order (g, m) such that

$$SB = Z - E \mid \varepsilon^2 = \text{trag}\,(E^tE) = \text{minimum}.$$

Naturally, it is a special case of the multivariate regression problem, so the solution is easily obtained by differentiating the function

$$\begin{aligned} f(B) &= \text{trag}\,((Z - SB)^t(Z - SB)) \\ &= \text{trag}\,(R) - \text{trag}\,(B^tS^tZ) - \text{trag}\,(Z^tSB) + \text{trag}\,(B^tS^tSB) \end{aligned}$$

by the elements of B matrix.

Since $\text{trag}\,(B^tS^tZ) = \text{trag}\,(Z^tSB)$ i $\text{trag}\,(R) = m$,

$$\partial f(B) / \partial B = - 2S^tZ + 2S^tSB,$$

and after dividing by 2 and reducing to zero,

$$S^tSB = S^tZ;$$

and since, certainly, S^tS is a regular diagonal matrix,

$$B = (S^tS)^{-1} S^tZ$$

so that it is obvious that the elements of the matrix

$$G = PZ = SB = (g_{ij})$$

10 Basically similar but formally different reformulation of canonical discriminant analysis defined in way that it is not immediately clear that this is about the method suggested by Anderson (1984).

$$i = 1,...,n; j = 1,...,m,$$

$$g_{ij} = (s_p{}^t s)^{-1} s_p{}^t z_j \mid e_i \in w_p$$

$$i = 1,...,n; j = 1,...,m,$$

are therefore the arithmetic mean of normalized and standardized variables in the subsamples that the objects from E belong to.

That is why the canonical discriminant analysis may be defined as a solution of the canonical problem

$$Zx_k = k_k, Gy_k = l_k \mid \rho_k = k_k{}^t l_k = \text{maximum}, k_k{}^t k_k = l_k{}^t l_k = \delta_{kq}, k_k{}^t l_q = 0 \mid k \neq q$$

$$k = 1,...,s; s = \min((g - 1), m)$$

where δ_{kq} is Kroneker's symbol and x_k and y_k are unknown m - dimensional vectors.

Since $\rho_k = x_k{}^t A y_k$, $k_k{}^t k_k = x_k{}^t R x_k$ and $l_k{}^t l_k = y_k{}^t A y_k$, for k = 1 the function to be maximized is

$$f(x_k, y_k, \lambda_k, \eta_k) = x_k{}^t A y_k - 2^{-1}\lambda_k(x_k{}^t R x_k - 1) - 2^{-1}\eta_k(y_k{}^t A y_k - 1).$$

By differentiating this function by the elements of x_k vector

$$\partial f/\partial x_k = Ay_k - \lambda_k R x_k,$$

and by differentiating by the elements of the vector y_k

$$\partial f/\partial y_k = Ax_k - \lambda_k A y_k.$$

After equalling to zero

$$Ay_k = \lambda_k R x_k$$

and

$$Ax_k = \lambda_k A y_k.$$

By differentiating by λ_k and η_k it is easy to obtain, from the condition $x_k{}^t R x_k = 1$ and $y_k{}^t A y_k = 1$, that $\lambda_k = \eta_k$. By multiplying the first result by R^{-1}

$$x_k \lambda_k = R^{-1} A y_k$$

then

$$x_k = R^{-1} A y_k \lambda_k^{-1}.$$

According to the second result

$$Ax_k \lambda_k^{-1} = Ay_k$$

so that

$$y_k = x_k\lambda_k^{-1}.$$

Therefore,

$$R^{-1}Ax_k\lambda_k^{-1} = x_k\lambda_k;$$

so by multiplying this result by λ_k

$$R^{-1}Ax_k = x_k\lambda_k^{2},$$

is obtained so that the problem is reduced to solving the general problem of eigenvalues

$$(R^{-1}A - \lambda_k I)x_k = 0,$$

$$k = 1,..., s$$

respectively

$$(A - \lambda_k R)x_k = 0$$

$$k = 1,..., s$$

and

$$\rho_k = x_k^{t}Ay_k = x_k^{t}Ax_k\lambda_k^{-1} = \lambda_k,$$

$$k = 1,..., s$$

are canonical correlations between the linear combinations of the variables from Z and G which are proportional to the differentiation of the centroids of the subsamples defined by the selection matrix S in the space stretched by the vectors from the variables from Z.

Like all the other statistical methods that are special cases of canonical correlation analysis, canonical discriminant analysis is invariant to any nonsingular transformation of the variables, therefore it is also metrically invariant.

Let H be any nonsingular matrix of order (m), let

$$Z_h = ZH$$

and let

$$G_h = PZ_h.$$

Then

$$R_h = Z_h^{t}Z_h = H^{t}RH,$$

$$A_h = G_h^{t}G_h = Z_h^{t}G_h = H^{t}AH,$$

and, since the matrices $R^{-1}A$ and $H^{-1}R^{-1}AH$ are similar, the problem is reduced to solving the characteristic equation

$$(H^{-1}R^{-1}AH - \lambda_k I)H^{-1}x_k = 0$$

so it is evident that the discriminant functions k_k and canonical correlations ρ_k are really invariant to the metrics of the variables from V.

Let $\rho = (\rho_k)$, k = 1,..., s be a diagonal matrix whose elements are canonical correlations, let $X = (x_k)$ and $Y = (y_k) = X\rho^{-1}$, k = 1,..., s be matrices of eigenvectors obtained by solving the canonical discriminant problem. Let

$$K = ZX$$

be a matrix of discriminant functions and let

$$L = GY = PZX\rho^{-1}$$

be a matrix of discriminant functions projected into a hypercube defined by the vectors of S matrix set to 1 after that projection. Obviously,

$$K^tL = X^tAX\rho^{-1} = X^tAY = \rho$$

since, of course, $K^tK = I$ and $L^tL = I$, are canonical discriminant analysis that produces two biorthogonal sets of vectors of the variables by such a transformation of the vectors of the variables from Z and G that it orthogonalizes those vectors and maximizes the cosines of the angles between the corresponding vectors from K and L with the additional condition that the cosines of the angles between the noncorresponding vectors from K and L are equal to zero.

However, that transformation maximizes, simultaneously, Euklidean distances between the centroids of the subsamples Ep from the sample E determined by the values on the nominal variable W on the discriminant functions from K. Let

$$M = (S^tS)^{-1}S^tK = BZX = (\mu_{pk})$$

$$p = 1,..., g;\ k = 1,..., s$$

be a matrix of the centroid of the subsample E_p on the discriminant functions, and let e_g be a summing vector of order g. Since $Z^te = 0$, then also $M^te_g = 0$, so the diagonal elements of the matrix

$$D^2 = \text{diag}\,(M^tM) = \text{diag}\,(X^tZ^tS(S^tS)^{-2}S^tZX)$$

are equal to the sum of the squares of the Euklidean distances between the elements μ_p of the vector μ_k from M. Let

$$\Omega = (S^tS)^{1/2}M = (\omega_{pk}) = (n_p^{1/2}\mu_{pk})$$

$$p = 1,..., g;\ k = 1,..., s;$$

obviously,

$$\Omega^t\Omega = X^tZ^tS(S^tS)^{-1}S^tZX = X^tAX = \rho^2,$$

and since $\rho_k{}^2$ = maximum $\forall\rho_k{}^2$, $k = 1,..., s \mid x_k{}^tRx_q = y_k{}^tAy_q = \delta_{kq}$, the maximization of the coefficients of correlation between the canonical variables from K and L which is equivalent to the maximization of the distances between the centroids of the subsample E_p on the discriminant functions.

Vectors x_k from X are, evidently, vectors of standardized partial regression coefficients of the variables from Z which generate the discriminant functions k_k that together with the discriminant functions l_k, formed by the vectors of the standardized partial regression coefficients $y_k = x_k\rho_k{}^{-1}$ from the variables from G, have the maximal correlations. Therefore the vectors x_k are proportional to the coordinates of the vectors of the discriminant functions in the oblique coordinate system that is composed of the vectors from Z with the cosines of the angles between the coordinate axes which are equal to the elements of the correlation matrix R. Hence the interpretation of the discriminant functions based on the set of those vectors is very complicated if the number of the variables from V is large enough to make the set V a satisfactorily representative sample from the set U.[11] Since the discriminant analysis may be interpreted as a special case of component analysis with the principal components transformed, by a permissible singular transformation, to maximize the distances between the centroids of the subsamples E_p, that is canonical correlations ρ_k (Cooley and Lohnes, 1971; Mulaik, 1972; Hadžigalić, 1984; Momirović and Dobrić, 1984; Hadžigalić, Bogdanović, Tenjović and Wolf, 1994), Cooley and Lohnes were, probably the first to suggest that the identification of the discriminant functions content should be based on the structural vectors f_k from the matrix

$$F = Z^tK = RX = (f_k) = (Rx_k),$$

analogous to the identification of the content of the canonical variables obtained by Hotelling's method of biorthogonal canonical correlation analysis. Since the elements f_{jk} of the matrix F behave as the regular product-moment correlation coefficients, and since they are the function of the normally distributed variables, therefore they are asymptotically normally distributed, their asymptotic variances are, of course,

$$\sigma_{jk}{}^2 \sim (1 - \phi_{jk}{}^2)^2n^{-1}$$

$$j = 1,..., m;\ k = 1,..., s$$

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11 A special problem is to test the hypothesis about the elements of the vector x_p, because so far no acceptable procedure for the evaluation of the matrices of the covariances of those elements has been proposed, except in the case of $g = 2$, because only then the canonical discriminant analysis could be treated as a special case of the regression analysis; then the matrix of the covariances of the elements of the single vector x is, certainly, $C_x = (1 - \rho^2)R^{-1}(n - m - 1)^{-1}$ (Seber, 1977; Štalec, Momirović and Zakrajšek, 1983; Anderson, 1984).

and could be used for testing the hypothesis of type H_{jk}: $f_{jk} = \phi_{jk}$, where ϕ_{jk} are some hypothetical correlations between the variables from V and discriminant functions in the population P because the asymptotic distribution of the coefficients f_{jk} is

$$f(f_{jk}) \sim N(\phi_{jk}, \sigma_{jk}^2)$$

where N is the symbol for normal distribution.

Momirović and Zorić (1996) also suggested the inspection of the cross-structural vectors c_k from the matrix

$$C = Z^tL = AY = RX\rho = F\rho$$

thus the factor matrix of A matrix, because, since $XX^t = R^{-1}$,

$$CC^t = RX\rho^2X^tR = X\rho^2X^{-1} = A.$$

It may easily be shown that F is a factor matrix of R matrix. Let

$$\Delta^2 = \text{diag}\ (X^tX)$$

and let

$$V = X\Delta^{-1}.$$

Then

$$\Delta^{-2} = V^tRV$$

is a diagonal matrix of the standardized variances of discriminant functions and

$$FF^t = RXX^tR = RV\Delta^2V^tR.$$

If s = m,

$$\Delta^2 = V^{-1}R^{-1}V^{-t}$$

and

$$XX^t = R^{-1}$$

so that

$$FF^t = R.$$

If s < m,

$$FF^t = RV(V^tRV)^{-1}V^tR$$

which is a special case of the general Guttman theorem on the factorization of any squared symmetrical matrix of rank m with any matrix of rank r < m.

In canonical discriminant analysis the main, and usually the only, set of hypotheses related to the parameters of that model is the set

$$H_0 = \{\varphi_k = 0, k = 1,..., s\}$$

where φ_k are hypothetical values of the canonical correlations in the population P.[12]

The hypotheses of type

$$H_{0k}: \varphi_k = 0 \qquad k = 1,..., s$$

may, especially in the case of g = 2, therefore s = 1, be tested in several ways. Most implementations of the canonical discriminant analysis apply one function of Wilks' (Wilks, 1932; 1935; 1962) measure

$$\lambda_k = \Sigma_{t+1}{}^{s} \log_e (1 - \rho_{t+1}{}^2) \qquad k = t + 1, t = 0, 1,..., s - 1$$

proposed by Bartlett (1941), who found that under the hypothesis H_{0k}: $\varphi_k = 0$ the functions

$$\chi_k{}^2 = - (n - (m + g + 3)/2)\lambda_k \qquad k = 1,..., s$$

had, approximately, χ^2 distribution with

$$\nu_k = (m - k + 1)(g - k)$$

degrees of freedom.

However, a more sensitive test of the hypotheses H_{0k}: $\varphi_k = 0$ is a function derived according to the maximum likelihood criterion suggested by Rao (Rao, 1951; 1973; Momirović, Gredelj and Szirovicza, 1977; Anderson, 1984). Let

$$a = ((m^2(g - 1)^2 - 4)/(m^2 + (g - 1)^2 - 5))^{1/2},$$

and let

$$\nu_{1k} = (m - k + 1)(g - k)$$

12 Hypothesis H_{01}: $\varphi_1 = 0$, which simply means that the arithmetic mean of all the variables from V may not be different in subpopulations P_p, p = 1,..., g from P is a subject, for unclear reasons, of a special statistical method which is commonly called multivariate analysis of variance. Naturally, if m = 1, then it is about one-factor analysis of variance which evidently comes down to the test of the hypothesis $\varphi = \eta = 0$, where η is Fisher's coefficient of intergroup correlation of a single variable v and nominal variable W; if nevertheless g = 2, this is certainly about the t - test for differences between the arithmetic mean of two independent subpopulations, that is a test of the hypothesis whether the point biserial coefficient of the correlation $\varphi = \eta = \rho_{pb}$ between v and {w1, w2} equals zero. This author has to admit that it has never been clear to him why the special cases of canonical discriminant analysis, that can easily be described in several lines of a footnote, are treated as special methods and described in separate chapters in most statistical textbooks and so taught to unsuspecting students of mathematical or applied statistics.

and

$$\nu_{2k} = a((n - 1) - (m - g)/2 - (m - k + 1)(g - k) - 2)/2.$$

Then the functions

$$f_k = (1 - \lambda_k^{-a})\lambda_k^{-a}(\nu_{2k}/\nu_{1k})$$

$$k = 1,...,s$$

have, under H_{0k}: $\varphi_k = 0$, the Fisher – Snedekorov F distribution with ν_{1k} and ν_{2k} degrees of freedom.[13]

Of course, although the discriminant functions are orthogonal, neither the tests of type χ_k^2, nor the tests of type f_k are really independent (Anderson, 1984); besides, the results of those tests, especially of Bartlett's test which is most frequently applied, are not, even when large samples are involved, in the best accordance with the results of the tests like

$$z_k = \rho_k/\sigma_k$$

$$k = 1,...,s$$

that are based on the fact that canonical correlations also have the asymptotical normal distributions with the parameters φ_k and

$$\sigma_k^2 \sim (1 - \varphi_k^2)^2 n^{-1}$$

(Kendall and Stuart, 1976; Anderson, 1984).[14]

ABOUT HOW TO CALCULATE DISCRIMINANT FUNCTIONS AND THE STRUCTURE OF DISCRIMINANT FACTORS IN SOME STATISTICAL SOFTWARE PRODUCTS

Although, it is obvious that, the canonical discriminant analysis, from a mathematical point of view, a rather simple method, and from a statistical point of view not too much complicated, some programmers employed in factories of commercial statistical program products have discovered, apparently, the way

13 This test is installed in the CANDISC program of the SAS program system accompanied by four independent tests of hypothesis $\varphi_1 = 0$ suggested by Wilks, Lawley and Hotelling, Pillai and Bartlett, Nanda and Pillai and Roy. These tests are described by Anderson (1984, pp. 321-333) but here they will not specially be considered because the trivial hypothesis $\mu_{jp} = 0 \ \forall \ \mu_{jp}$; j = 1,..., m; p = 1,...,g is rarely of significant interest to the majority of those whose structure of data requires the application of canonical discriminant analysis.

14 Because canonical discriminant analysis invariant to an arbitrary affine transformation of the variables, it may be derived as a special case of the canonical analysis of correlation between the variables from Z and S (Hadžigalić, 1984) or as a special case of canonical analysis of the covariances between the variables from $M = ZR^{-1/2}$ and S (Hadžigalić, Bogdanović, Tenjović and Wolf, 1995). However, in that case the consequences of the reckless definition of the problem that is supposed to be solved and the incorrect definitions of the structure of discriminant factors would not be so immediately clear.

and manner to introduce unnecessary confusion, thus proving that Steinitz's theorem is relevant in all segments of the human population.

The main, but not the only, reason for that confusion is the fact that in most texts dedicated to the canonical discriminant analysis (Rao, 1948; 1952; 1968; 1973; Rao and Slater, 1949; Anderson, 1966; Bryan, 1951; 1975; Cooley and Lohnes, 1971; Ivanović, 1963; 1977; Kendall and Stuart, 1976; Kovačić, 1994; Momirović, Gredelj and Szirovicza, 1977; Romeder, 1973) that method is defined, in accordance with the model of the multivariate analysis of variance, as a problem solution

$$\xi_k^2 = (v_k^t A v_k)(v_k^t W v_k)^{-1} = \text{maximum} \mid (v_k^t W v_q) = \delta_{kq},\ k,q = 1,\dots,s$$

which is, after few simple algebraic manipulations, reduced to the solution of the general problem of eigenvalues

$$(A - \xi_k^2 W)v_k = 0 \qquad k = 1,\dots, s.$$

Let $V = (v_k)$ i $\xi^2 = (\xi_k^2)$, $k = 1,\dots, s$. Since

$$V^t W V = I,$$

then

$$V^t A V = V^t W V \xi^2 = \xi^2$$

and hence

$$V^t R V = I + \xi^2$$

so that

$$X = V(I + \xi^2)^{1/2}$$

and

$$\rho^2 = \xi^2 (I + \xi^2)^{-1}.$$

Although such a solution is formally equivalent to the solution of a discriminant problem under the canonical model, it may easily be shown that in marginal cases this leads to the fact that the problem could not be resolved at all, or that the solution is burdened with so many numerical problems that the final result must be completely doubtful.

Since

$$X^t R X = X^t (A + W) X = I,$$

$$X^t W X = I - \rho^2 = \Lambda_W = (\lambda_{Wk}) \qquad k = 1,\dots, s$$

where λ_{Wk} Wilks' measures are now associated with the discriminant functions k_k from K. But, since

$$G^tH = 0,$$

then, if $Z \rightarrow PZ$, that is when the subpopulations P_p from P are almost totally quantitatively different, and thus $\rho \rightarrow I$, $H \rightarrow 0 \Rightarrow W \rightarrow 0$ and the problem of canonical discriminant analysis, defined in an ordinary way[15], becomes unsolvable, or the solution is numerically incorrect because of the weak conditioning of W matrix.

But while the problem of model choice is a consequence of the habit of the programmers and users of the ready-made statistical software products, but also, unfortunately, some professional statisticians, to read the statistical texts with their fingers, the forming of the structural matrices of discriminant functions in most of the commercial statistical program products[16] is simply a consequence of lack of thinking.

Those programs, however, consider cross-correlations between the variables from H and K thus correlations between the discriminant functions and those components of the variables from Z which are not at all included in the formation of those functions to be the structure of canonical discriminant functions. Actually, since $\Lambda = \text{diag } W$ is variance matrix of the variables from H, those programs define a structure matrix as

$$U = \Lambda^{-1/2}H^tK = \Lambda^{-1/2}Z^tQZX = \Lambda^{-1/2}WX = \Lambda^{-1/2}(R - A)X = \Lambda^{-1/2}(F - AX)$$

so that those variables from V, in which the subpopulations P_p from P mostly differ, define the structure of discriminant factors most poorly. Certainly, in a marginal case when $Z \rightarrow PZ$, and therefore, of course, $\rho \rightarrow I$, $U \rightarrow 0$ which is such evident nonsense that it is simply unbelievable that the programmers, who are normally very intelligent, did not see it immediately while writing or testing their own programs.

15 That is, certainly, true for both multivariate and univariate analyses of variance, which is sufficient evidence that the classical treatment of those methods should be abandoned and they should be considered as special cases of canonical correlation analysis.

16 As usual, SAS is an exception; program CANDISC from that system calculates the structural matrix of discriminant functions correctly but probably in order to satisfy the needs of those who are accustomed to the solutions suggested by other more popular program products, it calculates that matrix en passant and in the same senseless manner as the analogue programs from the package like Statistica and SPSS conduct.

INSTEAD OF DISCUSSION: ABOUT WHAT SHOULD AND SHOULDN'T BE DONE

Actually, it is simpler to determine what shouldn't be done: one shouldn't apply the programs for canonical discriminant analysis within which a canonical problem is defined in the classical Bryan's manner and which calculate, just for that reason, the structure of discriminant factors on the basis of the correlations between the variables from which the factors that differentiate the group are partialized as well as the discriminant functions no matter how those functions have been calculated. This practically means that there is no sense, at least when it is about the problems that have to be resolved by canonical discriminant analysis or by any other special case of a canonical discriminant model, to apply any commercial statistical software package, except for SAS, and the programs for the discriminant analysis specially written in some statistical metalanguages, like GENSTAT and SS.[17]

If SAS were available, in every sense of that word, to the users of statistical software products, it would be easy to say what should be done: The canonical discriminant analysis should be performed by the program CANDISC from that system, or by the analogous programs written in SS or GENSTAT language. Unfortunately, SAS is not available to the unprofessional statisticians not only for administrative or economic reasons, but also for something more serious, and that concerns SS and even more it concerns GENSTAT: Those systems are not intended for the unprofessional statisticians, so a regular user does not know how to use them, because there is no time, nor prior knowledge, to learn how to use them; that is obvious from the fact that the majority avoids to apply them even though they have them, since in some way they have overcome the administrative and economic problems.

Accordingly, there is only one reasonable way out which is possible because SPSS is far by the most popular statistical package and there is still one, although very cumbersome, language in which it is possible to program in SPSS environment. That way out implies persuading or forcing someone to write a correct program for canonical discriminant analysis in the Matrix language and to implement it as an additional part of the SPSS syntax. The one, who will do it, will doubtlessly accomplish, a good deed, since the canonical discriminant

17 The program CANDID (Momirović, 1987), written in the SS language, calculates the discriminant functions in the Mahalanobis's space in the manner described in one later published work of Hadžigalić, Bogdanović, Tenjović and Wolf (1995) in order to avoid some numerical problems and to enable testing the significance of the discriminant coefficients. Similarly the algorithm is easily performed even with a few simple manipulations by the commands of the GENSTAT language, because the function for the canonical discriminant analysis, which behaves in that language as an elementary command, calculates the parameters of the discriminant model correctly.

analysis is a method without which serious research is impossible neither in any natural nor social science, even in any technological discipline derived from those sciences.

3. Results and discussion

In table 59 there are the eigenvalues (Sv. vre.), percentage of the explained intergroup variability (Proc. var.), canonical correlation coefficient (Kan. Kor.), Wilks' Lamda values (Lamda), values of Bartlet's chi test (Chi), degrees of freedom (DF), statistical significance (Sig), set of the discriminant functions of the motor variables (FUNC1, FUNC2) and centroids of the groups indicated by the discriminant functions (C1 i C2).

By transformation and condensation of the variables in the space of musical abilities only one discriminant function, which maximally separates the groups of athletes according to discriminant coefficients, has been isolated.

This discriminant function explains the differences with 100 percent of the intergroup variability in the space of musical abilities of the applied discriminant variables.

Examining the coefficients that determine the first discriminant function, it could be noticed that it separates the dancers on the basis of all the tests used for the evaluation of musical abilities except the test which estimates the pitch. On the basis of the value and sign of the projection of the centroids on the first discriminant function, it may be concluded that the female dancers have a more pronounced ability to recognize the duration of tone, memory, rhythm and the ability to register the timbre of the tone. The male dancers have a more developed sense of registering the volume of a tone that is receptive signal.

DISCRIMINANT ANALYSIS OF THE TESTS OF MUSICAL ABILITIES

Table 1.

Fen	1*
Eig.val.	.4579
Pct of Vari.	100.00
Cum. Pct.	100.00
Can. Cor.	.56
Wilks' Lambda	.68

Table continued on next page...

...Table continued from previous page	
Chi	95.19
DF	5
Sig	.00

** FUNCTION FUNC 1*

DUT	.78
MEM	.68
RIT	.25
JAT	-.24
BOT	.23
VIT	.05

CENTROIDS OF THE GROUPS

GROUPS	FUNC1
FEMALE DANCERS 1	.66
MALE DANCERS 2	-.68

4. Conclusion

The research was conducted with the aim to determine the differences in the structure of musical abilities of the male and female dancers involved in standard and Latin American dances.

In order to determine the differences in the structure of musical abilities of male and female dancers, 267 examinees were involved, aged from 11 to 13, who are actively engaged in standard and Latin American dances.

For estimating the musical abilities, the well-known Seashore test battery to evaluate musicality was used. This battery estimates the following tests: pitch discrimination test, tone volume discrimination test, test for recognizing the rhythm, test for discrimination of the duration of a tone, test for discrimination of the timbre of a tone and a tonal memory test.

All the data collected in this research were processed in the Multidiscipline Research Centre of the Faculty of Sports and Physical Education, University of Priština, supported by the system of data processing programs developed by Popović, D. (1980), (1993), Momirović, K. and Popović, D. (2003).

By applying transformation and condensation of the variables in the space of musical abilities, there was isolated only one discriminative function, which maximally classifies the groups of athletes on the basis of discriminant coefficients. Examination of the coefficients, that determine the first discriminant function, allows classification of the dancers according to nearly all the tests for estimating musical abilities except the tests for the evaluation of pitches. According to the value and sign of the projection of centroids on the first discriminant function, it could be concluded that female dancers have a more pronounced ability to recognize the volume of tones, memory, rhythm and, moreover, the ability to register the timbre of tones. The male dancers have a more developed sense of registering the volume of tones, that is, receptive signals.

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Received on Nov. 23rd, 2010.
Approved on Dec. 17th, 2011.

Original scientific paper *UDC 793.3.071.2:159.923.2.072.59*

Differences in the level of musical abilities of male and female dancers

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Summary

The research was conducted to determine the differences in the structure of musical abilities of female and male dancers engaged in standard and Latin American dances. For estimating the differences, 267 dancers aged from 11 to 13 were involved. For estimating musical abilities, the well-known Seashore test battery for the assessment of musicality was used. This battery evaluates the following tests: a test for pitch discrimination, a test for tone intensity discrimination, a test for the recognition of rhythm, a test for tone duration discrimination, a test for timbre discrimination and a tonal memory test. All the data collected in this research were processed in the Multidiscipline Research Centre of the Faculty of Sports and Physical Education, the University of Priština, supported by the system of data processing programs developed by D. Popović, 1980, 1993, K. Momirović and D. Popović 2003. By transformation and condensation of the variables in the space of musical abilities, there was isolated only one discriminative function, which maximally separates the groups of athletes on the basis of the *discriminative coefficients. Examination of the coefficients which determine the first discriminative function, allows the classification of the dancers according to almost all the tests for estimating musical abilities except the tests for the evaluation of pitches. Based on the value and sign of the projection of centroids on the first discriminative function, it could be concluded that female dancers have a more pronounced ability of recognition of the volume of tones, memory, rhythm and, moreover, the ability of registering the timbre of tones. The male dancers have a more developed sense of registering the volume of tones, that is receptive signals.*

Keywords: /signal/centroid/coefficient/separate/ability/tone/rhythm/timbre/pitch/memory/

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