

# Application of a quasi-canonical discriminant model in determining differences between groups of athletes

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## Abstract

Canonical discriminant analysis can now be defined as a solution of the quasi-canonical problem  $\mathbf{M}\mathbf{x}_k = \mathbf{k}_k$ ,  $\mathbf{G}\mathbf{y}_k = \mathbf{I}_k | \mathbf{c}_k = \mathbf{k}_k \mathbf{I}_k = \text{maximum}$ ,  $\mathbf{x}_k^t \mathbf{x}_q = \mathbf{y}_k^t \mathbf{y}_q = \delta_{kq}$   $k = 1, \dots, s$ ;  $s = \min((g - 1), m) = m$  where  $\delta_{kq}$  is the Kronecker symbol and  $\mathbf{x}_k$  and  $\mathbf{y}_k$  are unknown  $m$ - dimensional vectors.

As  $\mathbf{c}_k = \mathbf{x}_k^t \mathbf{A}\mathbf{y}_k$ , the function to be maximized is, for  $k = 1$ ,  $f(\mathbf{x}_k, \mathbf{y}_k, \lambda_k, \eta_k) = \mathbf{x}_k^t \mathbf{A}\mathbf{y}_k - 2^{-1} \lambda_k (\mathbf{x}_k^t \mathbf{x}_k - 1) - 2^{-1} \eta_k (\mathbf{y}_k^t \mathbf{y}_k - 1)$ .

After differentiating this function by elements of vector  $\mathbf{x}_k$ ,  $\partial f / \partial \mathbf{x}_k = \mathbf{A}\mathbf{y}_k - \lambda_k \mathbf{x}_k$ , and after differentiating it by elements of vector  $\mathbf{y}_k$ ,  $\partial f / \partial \mathbf{y}_k = \mathbf{A}\mathbf{x}_k - \eta_k \mathbf{y}_k$ ; after equalizing with zero,  $\mathbf{A}\mathbf{y}_k = \lambda_k \mathbf{x}_k$  and  $\mathbf{A}\mathbf{x}_k = \eta_k \mathbf{y}_k$ . Through differentiating by  $\lambda_k$  and  $\eta_k$ ,

it is easily obtained, from the conditions  $\mathbf{x}_k^t \mathbf{x}_k = 1$  and  $\mathbf{y}_k^t \mathbf{y}_k = 1$ , that  $\lambda_k = \eta_k$ . As  $\mathbf{A}^t = \mathbf{A}$ , multiplying the first result by  $\mathbf{x}_k^t$  and the second result by  $\mathbf{y}_k^t$ ,  $\mathbf{x}_k^t \mathbf{A} \mathbf{y}_k = \lambda_k$  and  $\mathbf{y}_k^t \mathbf{A} \mathbf{x}_k = \lambda_k$ , so  $\mathbf{x}_k = \mathbf{y}_k$  and the problem comes down to an ordinary problem of eigenvalues and eigenvectors of matrix  $\mathbf{A}$ , that is the solution of the problem  $(\mathbf{A} - \lambda_k \mathbf{I}) \mathbf{x}_k = \mathbf{0}$ ,  $k = 1, \dots, m$ , so  $c_k = \rho_k^2 = \mathbf{x}_k^t \mathbf{A} \mathbf{x}_k = \lambda_k$ ,  $k = 1, \dots, m$  are squares of canonical correlations between the linear combinations of variables from  $\mathbf{M}$  and  $\mathbf{G}$  which are proportional to the differentiation of centroids of the subsamples defined by selector matrix  $\mathbf{S}$  in the space spanned by the vectors of variables from  $\mathbf{M}$ .

Let  $\boldsymbol{\rho}^2 = (\rho_k^2)$ ,  $k = 1, \dots, m$  be a diagonal matrix whose elements are squares of canonical correlations, let  $\mathbf{X} = (\mathbf{x}_k)$ ,  $k = 1, \dots, m$  be a matrix of eigenvectors obtained by solving the canonical discriminant problem, let  $\mathbf{K} = \mathbf{M} \mathbf{X}$  be a matrix of discriminant functions and let  $\mathbf{L} = \mathbf{G} \mathbf{X} = \mathbf{P} \mathbf{M} \mathbf{X}$  be a matrix of the discriminant functions projected into the hypercube defined by vectors of matrix  $\mathbf{S}$ . As  $\mathbf{K}^t \mathbf{L} = \mathbf{X}^t \mathbf{A} \mathbf{X} = \boldsymbol{\rho}^2$  and as, of course,  $\mathbf{K}^t \mathbf{K} = \mathbf{I}$  i  $\mathbf{L}^t \mathbf{L} = \boldsymbol{\rho}^2$ , the canonical discriminant analysis produces two biorthogonal sets of vectors of variables by such transformation of vectors of variables from  $\mathbf{M}$  and  $\mathbf{G}$  that orthogonalizes those vectors and maximizes cosines of the angles between corresponding vectors from  $\mathbf{K}$  and  $\mathbf{L}$ , with the additional condition that cosines of the angles of non-corresponding vectors from  $\mathbf{K}$  and  $\mathbf{L}$  are equal to zero, because the correlations between the variables from  $\mathbf{K}$  and  $\mathbf{L}$  are  $\mathbf{K}^t \mathbf{L} \boldsymbol{\rho}^{-1} = \mathbf{X}^t \mathbf{A} \mathbf{X} \boldsymbol{\rho}^{-1} = \boldsymbol{\rho}$ .

**Keywords:** / matrix / centroids / canonical / function / discriminant / vector /

## 1. Introduction

Dancing, as a form of human activity associated with music, represents a part of the rich tradition and artistic creativity of people. It is a part of people's spirit, perceptions and aspirations, the mirror of human life, thoughts and activities in general. Dancing originated with man, followed him throughout life and work and developed in accordance with the development of human society; at different levels of development it was changed, modified and enriched until it reached its final form as stylized artistic dance. Conceptually, it can be characterized as a structure of specific movement elements composed into a visible form through which the complexity of man's inner life is expressed.

Dance, first of all, expresses its creators' ideas through various structures of motions and movements as well as gestures, i.e. the dancer expresses the conceptions through his or her body activities. Dance is made up of freely thought-out or special structural movements composed into certain figures or units which alternate in the same or different order, at the same or different tempo and rhythm.

Motions and movements are mainly emphasized by the lower limbs, while the whole body follows the expression shaping the whole story into one unit. Dance elements have particularly made a major contribution to the improvement of movement coordination, formation of motor skills, development of movement memory, ear for music, rhythm and memory, contribution to physical development of functional abilities, increase of neuromuscular coordination, and, very significantly, associated with music or a song, they create an optimistic, joyful atmosphere, strengthen friendships and cooperation in a group, develop a sense of socialization and cooperation between the sexes, and represent an excellent tool in physical education.

These activities are very closely related to the aesthetic formation of personality through body training process carried out through pedagogical application of aesthetic regularities, or aesthetic education. The role of aesthetic education is to teach trainees to recognize the beauty in body training and sport activities which they will experience, express and creatively introduce in all areas of their lives.

The content of aesthetic education in a body training process is to form the beauty of the human body which implies proportional harmony, correct posture, body shaping, harmonious development of motor skills and physical properties, formation of knowledge, skills and agility, which are a prerequisite of the beauty of movement expression that means the unity of technical perfection and style when developing movement and motor skills during body training and sport activities, as well as development of sense of rhythm through expression of music by movements in daily locomotion.

## 2. Methods

### 2.1. Sample of respondents

The population from which the sample was taken for this research can be defined as a population of male and female dancers from folk ensembles of Serbia aged 18-25 years. Based on the posed problem, subject and aim of the research, and taking into account the organizational financial capabilities necessary for the implementation of the research procedure, an optimal number of respondents was taken into the sample in order to carry out the research correctly and obtain exact results. The sample of respondents consisted of 248 male and female dancers, members of folk ensembles of Serbia, which was an optimal number for the planned research.

The respondents met the following conditions:

- the age of respondents was defined on the basis of chronological age, so the respondents covered by the research were 18-25 years old

- they did not suffer from organic or somatic diseases
- they were active members of folk ensembles.

The research was conducted in the folk ensembles as follows: “Vuk St. Karadzic” from Backa Topola, “Svetozar Marković” from Novi Sad, “Zeleznicar”, “Vila” from Novi Sad, “Ravangrad” from Sombor, “Kosta Abrasevic” from Backa Palanka, “Stepino Kolo” from Stepanovicevo, “Taras Sevchenko” from Djurdjevo, “Kisac” from Kisac, “Sonja Marinkovic” from Novi Sad, “Soko” from Indjija.

## 2.2. Sample of variables of musicality

The measurement of musical abilities was performed using the Seashore test battery that assesses the basic musical abilities and contains the following components: pitch distinction (*PITCH*), loudness distinction (*LOUDNESS*), rhythm memory (*RHYTHM*), tone duration distinction (*TIME*), tone timbre distinction (*TIMBRE*), tone memory (*MEMORY*).

## 2.3. Data processing methods

There is no researcher nowadays who during his or her career years did not use, at least once, a model of multivariate analysis without understanding its logic. Therefore, the problem of selecting certain data processing methods is rather complex. That is why the value of a research depends not only on the sample of respondents and sample of variables, that is, the values of basic information, but also on the applied procedures for transformation and condensation of such information. Some scientific problems can be solved with the help of a number of different, and sometimes equally valuable, methods. However, with the same basic data, different conclusions can be drawn from the results of different methods.

In order to come to satisfactory scientific solutions, the researchers used, first of all, correct and then adequate, impartial and comparable procedures conforming to the nature of the posed problem and allowing extraction and transformation of the appropriate dimensions, testing of hypotheses on those dimensions, determination of differences and regularities within the research area.

Taking this into account, the methods that correspond to the nature of the problem and do not leave too heavy restrictions on the basic information were selected for the purpose of this research. To determine differences between the groups, a method of quasi-canonical discriminant analysis was applied.

In this research, all the data were processed at the Multidisciplinary Research Center, Faculty of Sport and Physical Education, University of Pristina, through the

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In this research, all the data were processed at the Multidisciplinary Research Center, Faculty of Sport and Physical Education, University of Pristina, through the

system of data processing programs developed by Popovic, D. (1980), (1993) and Momirovic, K. & Popovic, D. (2003).

## Quqsi-canonical discriminant analysis

Canonical discriminant analysis can now be defined as a solution of the quasi-canonical problem  $\mathbf{M}\mathbf{x}_k = \mathbf{k}_k$ ,  $\mathbf{G}\mathbf{y}_k = \mathbf{l}_k$  |  $\mathbf{c}_k = \mathbf{k}_k^t \mathbf{l}_k = \text{maximum}$ ,  $\mathbf{x}_k^t \mathbf{x}_q = \mathbf{y}_k^t \mathbf{y}_q = \delta_{kq}$   $k = 1, \dots, s$ ;  $s = \min((g - 1), m) = m$  where  $\delta_{kq}$  is the Kronecker symbol and  $\mathbf{x}_k$  and  $\mathbf{y}_k$  are unknown  $m$ -dimensional vectors.

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Vectors  $\mathbf{x}_k$  from  $\mathbf{X}$  are, obviously, the vectors of standardized partial regression coefficients of variables from  $\mathbf{M}$  that generate discriminant functions  $\mathbf{k}_k$  which, together with discriminant functions  $\mathbf{l}_k$  formed by vectors of standardized partial regression coefficients  $\mathbf{x}_k$  from variables of  $\mathbf{G}$ , have maximum correlations. But as  $\mathbf{M}^t \mathbf{K} = \mathbf{X}$ , the elements of matrix  $\mathbf{X}$  are, at the same time, correlations of variables from  $\mathbf{M}$  and discriminant variables from  $\mathbf{K}$ , that, unlike the standard canonical discriminant model, allows easy testing of hypotheses on partial impact

of variables on the formation of discriminant functions. To identify discriminant functions, the elements of a cross structural matrix defined as correlations between variables from  $\mathbf{M}$  and  $\mathbf{L}$ , that is, the elements of matrix  $\mathbf{Y} = \mathbf{M}'\mathbf{L}\boldsymbol{\rho}^{-1} = \mathbf{A}\mathbf{X}\boldsymbol{\rho}^{-1} = \mathbf{X}\boldsymbol{\rho}$ , can also be of certain significance; note, by the way, that  $\mathbf{Y}$  is a factor matrix of matrix  $\mathbf{A}$ , as, of course,  $\mathbf{Y}\mathbf{Y}' = \mathbf{X}\boldsymbol{\rho}^2\mathbf{X}'$ .

As elements  $x_{jk}$  of matrix  $\mathbf{X}$  and elements  $y_{jk}$  of matrix  $\mathbf{Y}$  are ordinary correlations, their asymptotic variances are  $\sigma_{x_{jk}}^2 = (1 - x_{jk}^2)^2 n^{-1}$ , respectively,  $\sigma_{y_{jk}}^2 = (1 - y_{jk}^2)^2 n^{-1}$ , so the hypotheses of type  $H_{0x_{jk}}$ , or  $H_{0y_{jk}}$ , can be tested based on the functions  $f_{x_{jk}} = x_{jk}^2((n - 2)(1 - x_{jk}^2))$ , respectively,  $f_{y_{jk}} = y_{jk}^2((n - 2)(1 - y_{jk}^2))$ , because under the hypotheses, these functions have the Fisher-Snedecor  $F$ -distribution with the degrees of freedom of  $v_1 = 1$  and  $v_2 = n - 2$ .

Unfortunately, with a usual application of canonical discriminant analysis, the main, and often the only, set of hypotheses related to the parameters of that model is the set  $H_0 = \{\phi_k = 0, k = 1, \dots, m\}$  where  $\phi_k$  are hypothetical values of canonical correlations in population  $P$ .

To test the hypothesis of type  $H_{0k}: \phi_k = 0, k = 1, \dots, m$ , researchers usually apply the function of the known Wilks measure  $\lambda_k = \sum_{t+1}^s \log_e (1 - \rho_{t+1}^2), k = t + 1, t = 0, 1, \dots, m - 1$  proposed by Bartlett (1941) who found that under the hypothesis  $H_{0k}: \phi_k = 0$ , functions  $\chi_k^2 = -(n - (m + g + 3)/2) \lambda_k, k = 1, \dots, m$  have, approximately,  $\chi^2$  distribution with the  $v_k = (m - k + 1)(g - k)$  degree of freedom.

However, the results of Bartlett's test are not, even when dealing with large samples, in full accordance with the results of the tests of type  $z_k = \rho_k / \sigma_k, k = 1, \dots, s$  which are based on the fact that canonical correlations have also asymptotically normal distributions with parameters  $\varphi_k$  and  $\sigma_k^2 \sim (1 - \varphi_k^2)^2 n^{-1}$ , (Kendall & Stuart, 1968; Anderson, 1984).

The centroids of subsamples  $E_p, p = 1, \dots, g$  from  $E$  on the discriminant functions necessary to identify the content of the discriminant functions, are, of course, the elements of matrix  $\mathbf{C} = (\mathbf{STS})^{-1}\mathbf{StK} = (\mathbf{STS})^{-1}\mathbf{StMX} = (\mathbf{StS})^{-1}\mathbf{StZR} - 1 / 2\mathbf{X}$ , and it is clear that they are, in fact, the centroids of the subsamples on the variables transformed into a Mahalanobis form projected into the discriminant space.

### 3. Discussion

As previously stated, musical abilities are a special "factor", or a special type of intelligence.

Modern studies have found that music stimulates not only one "music center", but a number of them. The brain processes component by component, with the help of specific neural circuits. It handles tone pitch, intensity, duration, and timbre. Higher

brain centers bring this information together creating a melody, rhythm, tempo, meter, and, ultimately, phrases and whole compositions in the representation.

The pitch, sounds and timbre of different instruments, as well as tempo, rhythm, loudness are processed in different parts of the brain. For higher cognitive functions such as listening to music, musical attention, musical memory, tracking tone lines, harmonies and rhythm patterns, as well as tracking harmonic structure and musical form structure, particular neural processing networks are built in the brain.

The results of discriminant analysis in the space of musicality are shown in Tables 1, 2, 3 and 4. Through analysis, it can be determined by that a significant canonical correlation of (.32) is obtained, and it explains 100% of valid variance of the entire system of the space being evaluated.

Female dancers have a better ability to distinguish tone duration, while male dancers have a better ability to distinguish loudness, pitch, rhythm, they have better overall musicality (as in the research conducted by Boli, E., Popovic, D., Popovic, J., 2012).

It is very important to observe once more the elements of music, or the fact that rhythm, where the male dancers achieved better results (variable RHYTHM), is defined as a sequence of tones or sounds of unequal duration, where female dancers achieved better results (variable TIME). The male dancers demonstrated a better sense of loudness which is a key factor when determining the meter, as well as rhythm (male dancers had better results in the rhythm assessment), which is quite logical because rhythm and meter are inseparable.

Compared to the research conducted by Boli, E. (2011) with the same measuring instruments but on a sample of younger respondents (11-13 years of age) engaged in another dance type (ballroom dancing - Latin and standard), the results also indicate that female dancers have a better ability to distinguish tone duration (as in that research) and male dancers also have a better ability to distinguish loudness (as in that research). The difference is that female performers of Latin and standard dances have a better ability of tone memory, rhythm recognition and timbre perception. In this research, male dancers generally achieve better results. The cause can be sought in a completely different conception of folk, on the one hand, and ballroom (Latin and standard) dances, on the other hand. Namely, a female ballroom dancer plays a dominant or equal role in the performance of dance structures, and a male ballroom dancer has a supporting role, while male folk dancers, compared to female folk dancers, perform motor, and thus rhythmically more complicated, dance structures.

## DISCRIMINANT ANALYSIS OF VARIABLES OF MUSICALITY

Table 1

Function	Eigenvalues	Variance %	Cumulative %	Can. R	Wilks Lambda	Chi-skor	df	Sig
1	.09	100.0	100.0	.32	.95	21.44	6	.05

## MATRIX M

Table 2

	FUNCTION 1
PITCH	.51
RHYTHM	.29
TIME	-.84
TIMRE	.07
MEMORY	.20

## STRUCTURE OF VARIABLES OF MUSICALITY

Table 3

	FUNCTION 1
TIME	-.52
LOUDNESS	.43
PITCH	.42
RHYTHM	.24
OVERALL MUS.	.23
MEMORY	.16
TIMBRE	.13

## CENTROIDS OF THE GROUPS

Table 4

N.G.	CEN1
female dancers	-.18
male dancers	.26

## 4. Conclusion

The research was conducted in order to determine differences in the structure of musical dimensions of male and female folk dancers.

To determine differences in the structure of musical dimensions of male and female folk dancers, 103 male and 145 female dancers aged 18 to 28 years, who were actively engaged in folk dancing, were tested.

For the assessment of musical abilities, the researchers used the Seashore test battery that assesses the basic musical abilities and contains the following components: pitch distinction, loudness distinction, rhythm memory, tone duration distinction, timbre distinction, and tone memory.

All the data in this research were processed at the Multidisciplinary Research Center, Faculty of Sport and Physical Education, University of Pristina, through the system of data processing programs developed by Popovic, D. (1980), (1993) and Momirovic, K. & Popovic, D. (2003).

To determine differences between the groups, a method of discriminant analysis was applied.

The results of discriminant analysis in the space of musicality are shown in Tables 1, 2, 3 and 4. Through analysis, it can be determined that a significant canonical correlation of (.32) is obtained, and it explains 100% of valid variance of the overall system of the space being evaluated.

Female dancers have a better ability to distinguish tone duration, and male dancers have a better ability to distinguish tone intensity, pitch, rhythm and overall musicality.

It is very important to observe once more the elements of music, or the fact that rhythm, where the male dancers achieved better results (variable RHYTHM), is defined as a sequence of tones or sounds of unequal duration, where the female dancers achieved better results (variable TIME). The male dancers demonstrated a better sense of loudness which is a key factor when determining the meter, as well as rhythm (male dancers had better results in the rhythm assessment), which is quite logical as rhythm and meter are inseparable.

Compared to the research conducted by Boli, E. (2011) with the same measuring instruments but on a sample of younger respondents (11-13 years of age) engaged in another dance type (ballroom dancing - Latin and standard), the results also indicate that female dancers have a better ability to distinguish tone duration (as in that research) and male dancers also have a better ability to distinguish loudness (as in that research). The difference is that female performers of Latin and standard dances have a better ability of tone memory, rhythm recognition and timbre perception. In this research male dancers generally achieve better results.

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## *Primena jednog kvazi kanoničkog diskriminativnog modela u utvrđivanju razlika među grupama sportista*

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### *Sažetak*

*Kanonička diskriminativna analiza može se sada definisati kao rešenje kvazi kanoničkog problema  $Mx_k = k_k$ ,  $Gy_k = l_k$  |  $c_k = k_k$ ,  $l_k = \text{maximum}$ ,  $x_k^t x_q = y_k^t y_q = \delta_{kq}$ ,  $k = 1, \dots, s$ ;  $s = \min((g - 1), m) = m$  gde je  $\delta_{kq}$  Kronekerov simbol a  $x_k$  i  $y_k$  nepoznati  $m$ -dimenzionalni vektori.*

*Kako je  $c_k = x_k^t A y_k$ , funkcija koju treba maksimizirati je, za  $k = 1$   $f(x_k, y_k, \lambda_k, \eta_k) = x_k^t A y_k - 2^{-1} \lambda_k (x_k^t x_k - 1) - 2^{-1} \eta_k (y_k^t y_k - 1)$ .*

*Diferenciranjem ove funkcije po elementima vektora  $x_k$   $\partial f / \partial x_k = A y_k - \lambda_k x_k$ , a diferenciranjem po elementima vektora  $y_k$   $\partial f / \partial y_k = A x_k - \eta_k y_k$ ; nakon izjednačavanja*

sa nulom  $Ay_k = \lambda_k x_k$  i  $Ax_k = \eta_k y_k$ . Diferenciranjem po  $\lambda_k$  i  $\eta_k$  lako se dobija, iz uslova  $x_k^t x_k = 1$  i  $y_k^t y_k = 1$ , da je  $\lambda_k = \eta_k$ . Kako je  $A^t = A$ , množenjem prvog rezultata sa  $x_k^t$  i drugog rezultata sa  $y_k^t$   $x_k^t A y_k = \lambda_k$  i  $y_k^t A x_k = \lambda_k$  pa je  $x_k = y_k$  i problem se svodi na običan problem svojstvenih vrednosti i vektora matrice  $A$ , dakle na rešenje problema  $(A - \lambda_k I)x_k = 0$ ,  $k = 1, \dots, m$  pa su  $c_k = \rho_k^2 = x_k^t A x_k = \lambda_k$ ,  $k = 1, \dots, m$  kvadrati kanoničkih korelacija između linearnih kombinacija varijabli iz  $M$  i  $G$  koje su proporcionalne diferencijaciji centroida subuzoraka deriniranih selektorskom matricom  $S$  u prostoru koga razapinju vektori varijabli iz  $M$ .

Neka je  $\rho^2 = (\rho_k^2)$ ,  $k = 1, \dots, m$  dijagonalna matrica čiji su elementi kvadrati kanoničkih korelacija, neka je  $X = (x_k)$ ,  $k = 1, \dots, m$  matrica svojstvenih vektora dobijenih rešavanjem kanoničkog diskriminativnog problema, neka je  $K = MX$  matrica diskriminativnih funkcija i neka je  $L = GX = PMX$  matrica diskriminativnih funkcija projektovanih u hiperkub definisan vektorima matrice  $S$ . Kako je  $K^t L = X^t A X = \rho^2$  i kako je, naravno,  $K^t K = I$  i  $L^t L = \rho^2$ , kanonička diskriminativna analiza proizvodi dva biortogonalna skupa vektora varijabli takvom transformacijom vektora varijabli iz  $M$  i  $G$  koja ortogonizuje te vektore i maksimizira kosinuse uglova između korespondentnih vektora iz  $K$  i  $L$  uz dodatni uslov da su kosinusi uglova nekorespondentnih vektora iz  $K$  i  $L$  jednaki nuli, jer su korelacije između varijabli iz  $K$  i  $L$   $K^t L \rho^{-1} = X^t A X \rho^{-1} = \rho$ .

**Ključne reči:** / matrica / centroidi / kanonička / funkcija / diskriminativna / vektor /

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